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Multiple Scattering Technique in the Simulation of a Phase Doppler Analyzer

1 Introduction

Practical significance of techniques, allowing to simulate the signal of a phase-Doppler analyser (PDA) in the case when two or more particles interact with illuminating beams, is described by Sankar et al. /1/. Authors of the above paper have developed a fast multiple scattering model of a PDA signal based on the geometrical optics.

With the aim of a more widely studding of the PDA signal, we suggested to use a rigorous Mie-scattering technique. The base of this technique is an interactive one proposed by Hamid /2/ for the case when an ensemble of spheres interacts with a plane wave. We have adapted the method of Hamid for two or more Gaussian (or non Gaussian) beams with elliptical polarization. For this purpose we incorporated in our technique the method of angular spectrum of plane waves (PWS) /3/.

Hence, the electromagnetic field scattered by an ensemble of spheres to an observation point is a superposition of the fields which are scattered by this ensemble illuminated by each i-th PWS plane wave. The technique is valid for the far field as well for the near field of scattering. It is also usable to investigate the signal of a maladjusted PDA system /4/, and of a laser radar /3,5/.

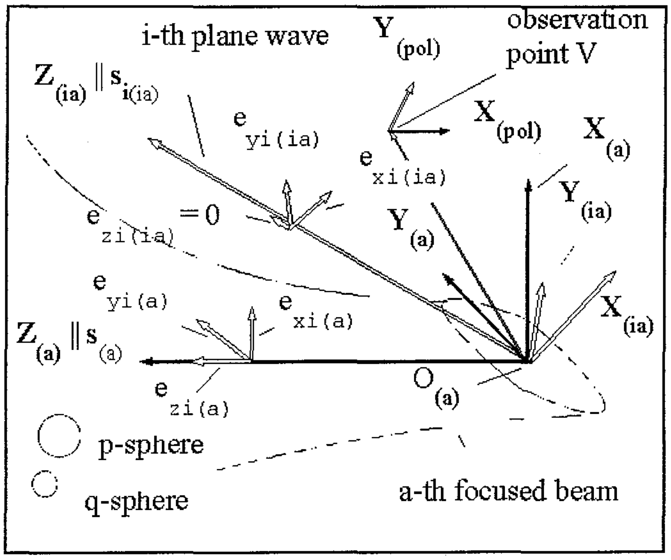
2 Theory

According to the principle of PWS, each a-th illuminating beam (Fig.1) can be expressed as a sum of homogeneous i-th plane waves, characterized in the coordinate system $XYZ_{(a)}$ of the illuminating beam by the propagation vector $\mathbf{s}_{i(a)}$, the unit polarization vector $\mathbf{e}_{i(a)}$ and the amplitude A_i /3,4/. The subscripts enclosed in brackets are used to indicate the corresponding coordinate system.

2.1 Expansion coefficients for the multiple scattered field in an arbitrary ξ-th coordinate system

It is known /2,6/ that iterative technique is effective one to define a scattered field when a plane wave interacts with an ensemble of p-th - q-th spheres (Fig. 2).

In an arbitrary $XYZ_{(\xi)}$ coordinate system propagation vector $\mathbf{s}_{i(\xi)}$ of the i-th plane wave of the a-th beam spectrum does not parallel to the axis $Z_{(\xi)}$.



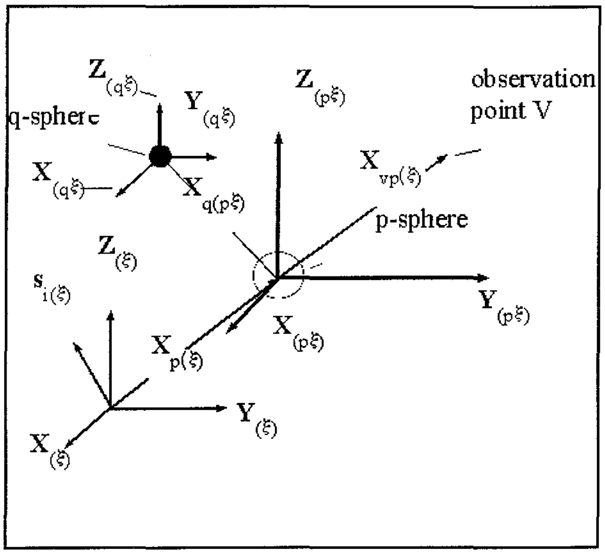


Fig.1: Interaction of an ensemble of moving spheres with a focused a-th beam and i-th plane wave of the spectrum of this beam. $\mathbf{e}_{ia}$ and $\mathbf{e}_{i(ia)}$ are the unit polarization vectors of the i-th plane wave in the $XYZ_{(a)}$ coordinate system of the a-th beam and in the coordinate system $XYZ_{(ia)}$ of the above plane wave. $XYZ_{(pol)}$ is the coordinate system of the receiver polarizer.

Fig.2: Multiple scattering in an arbitrary coordinate system $XYZ_{(\xi)}$.

The recurrent equation for the k-th iteration to simulate the expansion coefficient $a^{\Delta k}_{\varepsilon p\, mn}$ (or $b^{\Delta k}_{\varepsilon p\, mn}$) for the field scattered by the p-th sphere taking into account of the multiple scattering by other q-th spheres is described by Hamid /2/:

$$a^{\triangle k}_{\varepsilon\, p\, mn}(s_{q(p\xi)}, s_{i(\xi)}) = (-1)\alpha_n(\rho_p, m_p)\sum_{\nu=1}^{\infty}\sum_{\mu=-\nu}^{\nu}\Big[A^{\mu\nu}_{mn}(s_{q(p\xi)})\, a^{\triangle k-1}_{\varepsilon\, q\, \mu\nu}(s_{p(q\xi)}, s_{i(\xi)}) + B^{\mu\nu}_{mn}(s_{q(p\xi)})\, b^{\triangle k-1}_{\varepsilon\, q\, \mu\nu}(s_{p(q\xi)}, s_{i(\xi)})\Big] \quad , \tag{1}$$

where the corresponding coefficients for the first step of iteration (k=1) can be determine by the following expression, written in matrix form /7/:

$$[\boldsymbol{a}^{\triangle 1}_{\varepsilon p\; mn,c}(s_{i(\xi)})]_c = \begin{bmatrix} \boldsymbol{a}^{\triangle 1}_{\varepsilon p\; mn,c}(s_{i(\xi)}) \\ \boldsymbol{b}^{\triangle 1}_{\varepsilon p\; mn,c}(s_{i(\xi)}) \end{bmatrix} = \begin{bmatrix} \boldsymbol{\alpha}_{n,d}(\rho_p, m_p) & 0 \\ 0 & \boldsymbol{\beta}_{n,d}(\rho_p, m_p) \end{bmatrix} \begin{bmatrix} \boldsymbol{P}^{\Delta}_{\varepsilon\, mn,c}(s_{i(\xi)}) \\ \boldsymbol{Q}^{\Delta}_{\varepsilon\, mn,c}(s_{i(\xi)}) \end{bmatrix} . \tag{2}$$

For simplification, the equations for expansion coefficients $b^{k}_{\varepsilon p\, mn}$ are omitted in this paper. It is well known that for simulation of the coefficients $b^{k}_{\varepsilon\,p\,mn}$ we can use corresponding equations for the coefficients $a^{k}_{\varepsilon\,p\,mn}$ in which the letters "b" and "β" are taken instead the letters "a" and "α" (and vice versa). Subscripts d,c,r mean that a corresponding vector (matrix) is a diagonal, column or row vector (matrix); ε = x,y,z is the label of the corresponding element of the above vectors; m and n are the order and degree of the expansion coefficients; $\alpha_n(\rho_p,m_p)$ and $\beta_n(\rho_p,m_p)$ are the familiar Lorenz-Mie coefficients /2/; $A_{mn}^{\mu\nu}$ and $B_{mn}^{\mu\nu}$ are the translation addition theorem coefficients /2/; $\rho_p = k_a\, r_p$ is the Mie parameter; r_p and m_p are the radius and the complex refractive index of the p-th spherical particle; $\mathbf{s}_{p(q\xi)}$ is the position vector of the p-th sphere centre in the $XYZ_{(q\xi)}$ coordinate system of the q-th sphere; $\mathbf{s}_{q(p\xi)}$ is the position vector of the q-th sphere centre in the $XYZ_{(p\xi)}$ coordinate system (the corresponding axes of the $XYZ_{(\xi)}$, $XYZ_{(q\xi)}$ and $XYZ_{(p\xi)}$ coordinate system are parallel and the centres of $XYZ_{(q\xi)}$ and $XYZ_{(p\xi)}$ coincide with the centres of q-th and p-th spheres correspondingly). Superscript Δ shows that coefficients are modified by using the following equation /7/:

$$a^{\Delta\, k}_{\varepsilon\; p\; mn} = \frac{1}{k_P(\boldsymbol{s}_{i(a)},\boldsymbol{X}_{p(a)})}\, a^{k}_{\varepsilon\; p\; mn} \;, \tag{3}$$

where

$$k_P(\boldsymbol{s}_{i(a)},\boldsymbol{X}_{p(a)}) = k_{Pe}(\boldsymbol{s}_{i(a)},\boldsymbol{X}_{p(a)})k_{Pn} \;; \quad k_{Pe}(\boldsymbol{s}_{i(a)},\boldsymbol{X}_{p(a)}) = \exp(jk_a \boldsymbol{s}_{i(a)},\boldsymbol{X}_{p(a)}) \;; \tag{4}$$

$k_{Pn} = (j^{n})^{\alpha}$ is the coefficient with imaginary unit /7/; the coefficient α allows to simplify the calculations for the far scattered field; $\alpha = 0$ for the incident field and for the near- and far scattered field; $\alpha = 1$ only for the far scattered field ($k_a\, |\mathbf{X}_{pv}| \gg n^2$); $\mathbf{X}_{p(\xi)}$ is the position vector of the p-th sphere in the $XYZ_{(\xi)}$ coordinate system; $|\mathbf{X}_{pv}|$ is the distance between the p-th sphere and the V observation point.
It is evident that Eq. (1) for the determination of the scattered expansion coefficients is relative complicated. Our aim is to simplify this calculation.

2.2 Technique to simplify the calculation of the expansion coefficients for the multiple scattered field

A. Calculation in the $XYZ_{(qp\xi)}$ and in the $XYZ_{(pq\xi)}$ coordinate systems (translation rotation algorithm)

Mackowski /6/ presented the fast technique to calculate the above expansion coefficients though the tree steps.
Step 1. Transformation of the expansion coefficients form the $XYZ_{(p\xi)}$ coordinate system to the $XYZ_{(pq\xi)}$ coordinate system of p-th sphere or from the $XYZ_{(q\xi)}$ to the

$XYZ_{(qp\xi)}$ coordinate system of q-th sphere:

$$a_{\varepsilon\ p\ mn}^{\star\triangle\ k-1}(s_{q(pq\xi)},s_{i(\xi)}) = \sum_{l=-n}^{n} D_{mn}^{l}(\theta_{q(p\xi)})\Phi_l(\phi_{q(p\xi)})\ a_{\varepsilon\ p\ ln}^{\triangle\ k-1}(s_{q(pq\xi)},s_{i(\xi)}) \ , \tag{5}$$

where the superscript $\star$ denotes the calculations in the symmetrical coordinate system; $\Phi_l(\varphi) = \exp(jl\varphi)$.

As show in Fig. 3, the coordinate system $XYZ_{(pq\xi)}$ is formed by rotating $XYZ_{(p\xi)}$ through the angle $\varphi_{q(p\xi)}$ around axis $Z_{(p\xi)}$, and then though the angle $\theta_{q(p\xi)}$ around the $Y'_{(p\xi)}$; $\varphi_{q(p\xi)}$ and $\theta_{q(p\xi)}$ depend on the position vector $\mathbf{s}_{q(p\xi)}$ of q-th sphere in the coordinate system $XYZ_{(p\xi)}$.

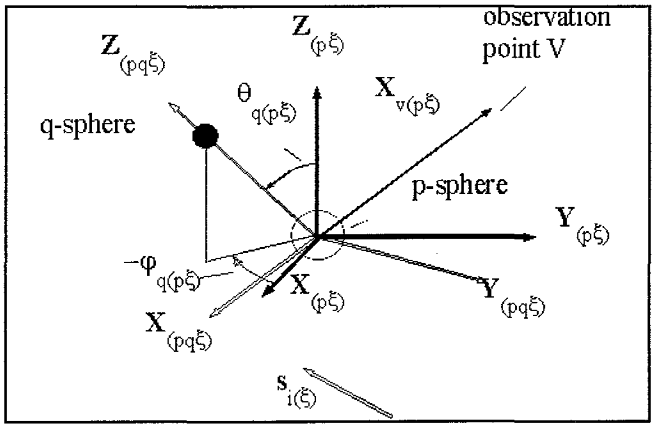


Fig.3: Arbitrary $XYZ_{(pq\xi)}$ coordinate system for the translation-rotation algorithm ($\mathbf{s}_{i(\xi)}$ does not parallel to the axis $Z_{(p\xi)}$).

In a similar manner the coordinate system $XYZ_{(qp\xi)}$ is formed by rotating $XYZ_{(q\xi)}$ through the angle $\varphi_{p(q\xi)}$ around axis $Z_{(q\xi)}$, and then though the angle $\theta_{p(q\xi)}$ around the $Y'_{(q\xi)}$; $\varphi_{p(q\xi)}$ and $\theta_{p(q\xi)}$ depend on the position vector $\mathbf{s}_{p(q\xi)}$ of p-th sphere in the coordinate system $XYZ_{(q\xi)}$.

Sep 2. Because p-th and q-th sphere are located on the $Z_{(qp\xi)}$ axes of the $XYZ_{(qp\xi)}$ coordinate system or on the $Z_{(pq\xi)}$ axes of the $XYZ_{(pq\xi)}$ coordinate system, the recurrent Eq. (1) may be sufficiently simplified /6/:

$$a_{\varepsilon\ p\ mn}^{\star\triangle\ k}(s_{q(pq\xi)},s_{i(\xi)}) = (-1)\alpha_n(\rho_p,m_p)\sum_{\nu=1}^{n_{\max}} [A_{mn}^{m\nu}(d_{pq})a_{\varepsilon\ q\ m\nu}^{\star\triangle\ k-1}(s_{p(qp\xi)},s_{i(\xi)}) + B_{mn}^{m\nu}(d_{pq})b_{\varepsilon\ q\ m\nu}^{\star\triangle\ k-1}(s_{p(qp\xi)},s_{i(\xi)})] \ , \tag{6}$$

where $n_{\max} = \rho_p + 4\rho_p^{1/3} + 2$ is the total number of terms required for a good convergence of Mie equations.

Step 3. The coefficients are rotated back to the original orientation through the following transformation:

$$a_{\varepsilon\ p\ mn}^{\triangle\ k}(s_{q(p\xi)},s_{i(\xi)}) = (-1)^m\Phi_m(-\varphi_{q(p\xi)})\sum_{l=-n}^{n} (-1)^l D_{mn}^{l}(\theta_{q(p\xi)})\ a_{\varepsilon\ p\ ln}^{\star\triangle\ k}(s_{q(pq\xi)},s_{i(\xi)}) \ , \tag{7}$$

where the transformation coefficients $D^1_{mn}(\theta_{q(p\xi)})$ are calculated by using the known recurrent relationships /6/. Mackowski showed that the above translation-rotation scheme will generally be $(1/3)n_{max}$ times faster then that accomplished through a pure solid translation equation (1).

B. Calculation in the symmetrical $XYZ_{(qpia)}$ and $XYZ_{(pqia)}$ coordinate systems

The main aim of this paper is the further simplification of the simulation of the expansion coefficients for the multiple scattered field. It is evident that for the symmetrical initial coordinate system $XYZ_{(ia)}$ (see Fig. 3 and Fig. 4 for ξ = ia, $Z_{ia} \parallel s_{ia}$) the following simplifications are valid for first iteration in the equation (2) of expansion coefficients /6/:

$$\begin{bmatrix} \overset{\triangle}{a}{}^{1}_{x\,p\,mn} \\ \overset{\triangle}{b}{}^{1}_{x\,p\,mn} \end{bmatrix} = \begin{cases} \begin{bmatrix} jD_{1n}\boldsymbol{\alpha}_n(\rho_p,m_p) \\ jD_{1n}\boldsymbol{\beta}_n(\rho_p,m_p) \end{bmatrix} \neq f(s_{q(p\xi)},s_{i(ia)}), & for \quad m = 1 \\ 0, & for \quad m \neq 1 \end{cases} \quad ; \tag{8}$$

$$\begin{bmatrix} \overset{\triangle}{a}{}^{1}_{y\,p\,mn} \\ \overset{\triangle}{b}{}^{1}_{y\,p\,mn} \end{bmatrix} = \begin{cases} \begin{bmatrix} -D_{1n}\boldsymbol{\alpha}_n(\rho_p,m_p) \\ -D_{1n}\boldsymbol{\beta}_n(\rho_p,m_p) \end{bmatrix} \neq f(s_{q(p\xi)},s_{i(ia)}), & for \quad m = 1 \\ 0, & for \quad m \neq 1 \end{cases} \quad , \tag{9}$$

and in the transformation equation (5)

$$\overset{\star\triangle}{a}{}^{1}_{\varepsilon\,p\,mn}(s_{q(pq\,ia)}) = \varepsilon_m D^1_{mn}(\theta_{q(pia)})\Phi_1(\phi_{q(pia)})\ \overset{\triangle}{a}{}^{1}_{\varepsilon\,p\,1n}(s_{q(pq\,ia)}) \ , \tag{10}$$

where $D_{1n} = (2n + 1)/(2n(n + 1))$,
Contribution of all q-th spheres to the scattering by p-th sphere can be estimate by the equation

$$\overset{\triangle}{a}{}^{k}_{p\,mn}(s_{i(ia)}) = \sum_{q=1,\ q\neq p}^{N_s} \overset{\triangle}{a}{}^{k}_{p\,mn}(s_{q(p\ ia)}) \quad . \tag{11}$$

2.3. PDA signal from an ensemble of moving spheres

A. Calculation of scattered factor in an arbitrary $XYZ_{(\xi)}$ coordinate system

Earlier /7/ we described the modified factor of the field scattered to an observation point in an arbitrary $XYZ_{(\xi)}$ coordinate system in the matrix form:

$$[\boldsymbol{F}^{\Delta}_{sp(\theta v\xi),c}(\boldsymbol{s}_{i(\xi)},\boldsymbol{s}_{pv(\xi)})]_r = [\boldsymbol{F}^{\Delta}_{xsp(\theta v\xi),c} \quad \boldsymbol{F}^{\Delta}_{ysp(\theta v\xi),c} \quad \boldsymbol{F}^{\Delta}_{zsp(\theta v\xi),c}] \;, \tag{12}$$

where

$$\boldsymbol{F}^{\Delta}_{\varepsilon sp(\theta v\xi),c}(\boldsymbol{s}_{i(\xi)},\boldsymbol{s}_{pv(\xi)}) = \begin{bmatrix} \boldsymbol{F}^{\Delta}_{\varepsilon\,\theta sp(\theta v\xi)m,c}(\boldsymbol{s}_{i(\xi)},\boldsymbol{s}_{pv(\xi)})\,\boldsymbol{\Phi}_{m,r}(\boldsymbol{s}_{pv(\xi)}) \\ \boldsymbol{F}^{\Delta}_{\varepsilon\,\varphi sp(\theta v\xi)m,c}(\boldsymbol{s}_{i(\xi)}\boldsymbol{s}_{pv(\xi)})\,\boldsymbol{\Phi}_{m,r}(\boldsymbol{s}_{pv(\xi)}) \\ \boldsymbol{F}^{\Delta}_{\varepsilon\,rsp(\theta v\xi)m,c}(\boldsymbol{s}_{i(\xi)}\boldsymbol{s}_{pv(\xi)})\,\boldsymbol{\Phi}_{m,r}(\boldsymbol{s}_{pv(\xi)}) \end{bmatrix} ; \tag{13}$$

$$\boldsymbol{\Phi}_{m,r}(\boldsymbol{s}_{pv(\xi)}) = [\; \varepsilon_{m_{\min}}\Phi_{m_{\min}} \quad \varepsilon_{m_{\min}+1}\Phi_{m_{\min}+1} \quad \dots \quad \varepsilon_{m_{\max}}\Phi_{m_{\max}} \;] \;; \tag{14}$$

$s_{pv(\xi)}$ is the unit vector of the direction from the p-th particle centre to the observation point V in the $XYZ_{(\xi)}$ coordinate system; $\Phi_m(\varphi_{pv(\xi)}) = \exp(jm\varphi_{pv(\xi)})$; ε_m is the symmetry coefficient of the summing over m ($\varepsilon_m = 1$, for the summing limits $\{m_{min} = -n_{max}; m_{max} = n_{max}\}$ and arbitrary m or for the $\{m_{min} = 0; m_{max} = n_{max}\}$ and $m = 0$; $\varepsilon_m = 2$, for the summing limits $\{m_{min} = 0; m_{max} = n_{max}\}$ and $m \geq 0$); $\{\boldsymbol{\theta}, \boldsymbol{\varphi}, \mathbf{r}\}$ are the orthonormal basis vectors described in /4/.

B. Calculation of the PDA signal in the symmetrical $XYZ_{(ia)}$ coordinate system

We showed also that $e_{zi(ia)}(s_{i(ia)}) = 0$ /4/ when initial coordinate system is XYZ_{ia} ($\xi = ia$) and the equation for a corresponding element of the modified scattering factor for the p-th sphere may be simulated by using relative simple expression /7/

$$F^{\Delta}_{\varepsilon\,\delta\,sp(\theta via)}(\boldsymbol{s}_{i(ia)},\boldsymbol{s}_{pv(ia)}) = \boldsymbol{\varepsilon}_m \sum_{n=1}^{n_{\max}} \sum_{m=-n}^{n} \sum_{k=1}^{k_{\max}} \{-j\,\boldsymbol{\tau}_{\delta\,mn}(\theta_{pv(ia)})\; a^{\Delta k}_{\varepsilon\,p\,,mn}(\boldsymbol{s}_{i(ia)}) - \boldsymbol{\pi}_{\delta\,mn}(\theta_{pv(ia)})\; b^{\Delta k}_{\varepsilon\,p\,mn}(\boldsymbol{s}_{i(ia)})\}\; \boldsymbol{\Phi}_m(\varphi_{pv(ia)})\} \quad , \tag{15}$$

where k_{max} is the optimal number of iterations /2/; $\tau_{\delta\,mn}$ and $\pi_{\delta\,mn}$ are the elements of the corresponding matrices τ_{mn} and π_{mn} described in /7/ and incorporating the associated Legender functions $P_n^m(\cos(\theta_{i(ia)})$.
This modified factor taking into account of the multiple scattering from N_s spheres illuminated by the i-th plane wave can be presented in the $XYZ_{(ia)}$ coordinate system by the equation

$$F^{\Delta}_{sav(\theta via)}(s_{i(ia)},s_{v(ia)},\beta t) = \sum_{p=1}^{N_s} k_{Ne2}(s_{v(a)},X_{p(a)})\; k_{Pe}(s_{i(a)},X_{p(a)}) \times F^{\Delta}_{sav(\theta via)}(s_{i(ia)},s_{pv(ia)})\; f^{\beta}_{sDia}(\omega_{Dia},\omega_{Da},t) \quad , \tag{16}$$

where

$$k_{Ne2}(s_{v(a)},X_{p(a)}) = \left[\exp(-j\,k_a s_{v(a)} X_{p(a)})\right]^{\alpha} \tag{17}$$

is the phase factor /7/; $f_{sDia}(\omega_{Dia},\omega_{Da},t) = f_{Dia}(\omega_{Dia}t)\cdot f^{-1}_{Da}(\omega_{Da}t)$; $f_{Dia}(\omega_{Dia}t) = \exp(+j\omega_{Dia}t)$ and $f_{Da}(\omega_{Da}t) = \exp(+j\omega_{Da}t)$ are the coefficients taking into account of the variation of the Doppler frequencies $\omega_{Dia} = k_a\mathbf{s}_{i(a)}\mathbf{V}_{p(a)} = k_a\mathbf{s}_{i(ia)}\mathbf{V}_{p(ia)}$ for the i-th plane wave and $\omega_{Da} = k_a\mathbf{s}_{a(0)}\mathbf{V}_{p(0)}$ for the a-th beam; $\omega_{sDa} = \omega_a - \omega_{Da}$ is the well known expression of the PDA signal frequency; k_a is the wave length of the a-th beam; γ and β are the coefficient allowing to simplify the calculations when $\mathbf{V}_p = \mathbf{V}_q$; $\mathbf{V}_p$ and $\mathbf{V}_q$ are the velocity vectors of the p-th and the q-th spheres; $\gamma = 0$ and $\beta = 1$, when $\mathbf{V}_p \neq \mathbf{V}_q$; $\gamma = 1$ and $\beta = 0$, when $\mathbf{V}_p = \mathbf{V}_q$.
Consequently, the electric vector $\mathbf{E}_{sav(pol)}(t)$ of the field scattered to the observation point V by an ensemble of particles taking into account the contribution of all i-th plane waves of the a-th beam angular spectrum can be written as:

$$E_{sav(pol)}(t) = A_a(|X_{(a)}| = 0)\; k_{Ne1}(|X_{v(a)}|)\; \tau_{pol}\; F^{\Delta}_{sav(pol)}(t) \ , \tag{18}$$

where

$$F^{\Delta}_{sav(pol)}(t) = R_{\Sigma}(s_{pol(\psi 0v0)},s_{v0(0)},s_{v(0a)})\cdot F^{\Delta}_{sav(\theta v0a)}(t) \quad ; \tag{19}$$

$$F^{\Delta}_{sav(\theta v0a)}(t) = f_{sDa}(\omega_{sDa}t) \times \iint F^{\Delta}_{sav(\theta v0a)}(s_{i(ia)},s_{v(ia)},\beta t)\, f^{\gamma}_{sDia}(\omega_{Dia},\omega_{Da},t)\, ds_{xi(a)}\, ds_{yi(a)} \ ; \tag{20}$$

$$F^{\Delta}_{sav(\theta v0a)}(s_{i(ia)},s_{v(ia)},\beta t) = R^{-1}_{\varphi}(-s_{i(\theta v0a)})\cdot F^{\Delta}_{sav(\theta via)}(s_{i(ia)},s_{v(ia)},\beta t) \quad ; \tag{21}$$

$\mathbf{R}_\Sigma$ and $\mathbf{R}_\varphi$ are the matrices /4/ to transform the scattered factor from $XYZ_{(ia)}$ coordinate system through $XYZ_{(0a)}$ to the $XYZ_{(pol)}$ coordinate system of receiver polarizer, which are valid both for adjusted and for maladjusted PDA system;

$$k_{Ne1}(|\boldsymbol{X}_{v(a)}|) = \left[j \, \frac{\exp(j k_a |\boldsymbol{X}_{v(a)}|)}{k_a \cdot |\boldsymbol{X}_{v(a)}|} \right]^\alpha \qquad \textbf{(22)}$$

is the phase factor of the modified coefficient for the vector spherical wave functions /7/ allowing to simplify calculations for the far field of scattering; τ_{pol} is the diagonal vector of the polarizer transmission coefficients. More detail matrices $\mathbf{R}_\Sigma$ and $\mathbf{R}_\varphi$, coordinate system $XYZ_{(0a)}$, unit vectors $\mathbf{s}_{v0(0)}$, $\mathbf{s}_{v(0a)}$, $\mathbf{s}_{i(\theta v0a)}$ and $\mathbf{s}_{pol(\psi 0v0)}$, and an interference field in the point V are described in /4/.
Hence, for each i-th plane wave of the a-th beam spectrum we have to simulate in the symmetrical coordinate system (see Fig. 3) only 4 sums

$$\boldsymbol{F}_{sp} = \sum_{n=1}^{n_{\max}} \sum_{m=-n}^{n} \sum_{q=1,\, q\neq p}^{N_s} \sum_{\nu=1}^{n_{\max}} f\,(A_{mn}^{m\nu}, B_{mn}^{m\nu}, \boldsymbol{a}_{q\;m\nu}) \qquad \textbf{(23)}$$

instead the five sums in the $XYZ_{(\xi)}$ coordinate system (see Fig. 2)

$$\boldsymbol{F}_{sp} = \sum_{n=1}^{n_{\max}} \sum_{m=-n}^{n} \sum_{q=1,\, q\neq p}^{N_s} \sum_{\nu=1}^{n_{\max}} \sum_{\mu=-\nu}^{\nu} f\,(A_{mn}^{\mu\nu}, B_{mn}^{\mu\nu}, \boldsymbol{a}_{q\;\mu\nu}) \quad . \qquad \textbf{(24)}$$

3 Summary

A fast technique to determine the PDA signal detected when an ensemble of arbitrary moving spheres interact with a few focused beams is presented. The technique is based on the rigorous Mie theory, translation-rotation scheme /6/ and on the fast technique to transform the electric vector of multiple scattered field from the symmetrical coordinate system to the coordinate system of the receiver polarizer. Simplification of this calculations is illustrated by the Eqs. (8) - (10), (23) and (24).

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5 Nomenclature

a, b expansion coefficients for the scattered field
E electric field vector
e unit and non-unit polarization vector, correspondingly
F scattering factor
k wave number
k_{Pe}, k_{Ne1}, k_{Ne2} phase factors
m, n order and degree of the vector spherical wave functions
n_{max} total number of terms required for good convergence of Mie equations
m_p complex refractive index of particle relative to the medium
PDA phase-Doppler anemometry (or analyzer)
PWS angular plane wave spectrum
R transformation matrix
s unit position or propagation vector
V observation point
α, β familiar Lorenz-Mie sphere coefficients
ε symmetry coefficient of the summation over order m
θ scattering angle
ρ_p Mie parameter of the p-sphere particle
τ_{pol} diagonal vector of the polarizer transmission coefficients
φ azimuth angle

Subscripts

a approaching wave
i i-th plane wave of angular spectrum
p,q p-th, q-th particles
s scattering
v observation point
θ, φ, ψ, μ labels of the coordinate system rotations
(ξ) label of the corresponding coordinate system

Superscripts

$\vartriangle$ modified matrix or vector
$\star$ denotes calculations in the symmetrical coordinate system

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